Further Maths GCSE
Trigonometry Answers

1. $\tan x = \frac{12-5\sqrt{3}}{4\sqrt{3}-5}$ rationalise denominator

$$\frac{(12-5\sqrt{3})}{(4\sqrt{3}-5)} \times \frac{(4\sqrt{3}+5)}{(4\sqrt{3}+5)} = \frac{48\sqrt{3}-25\sqrt{3}-60+60}{48-25}$$

$$= \frac{23\sqrt{3}}{23} = \sqrt{3}$$

$\tan x = \sqrt{3} \Rightarrow x = 60°$

2. Area of triangle $= \frac{1}{2}(4\sqrt{3}+6)(2-\sqrt{3})\sin A$

$= \frac{1}{2}[8\sqrt{3}+6\sqrt{3}-12-12] \times \frac{1}{4}$

$= \frac{1}{4}\sqrt{3}$

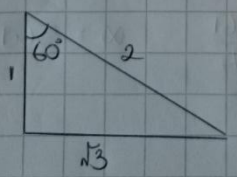
2. $\frac{\sin A}{2-\sqrt{3}} = \frac{\sin B}{4\sqrt{3}-6}$

$$\sin B = \frac{\frac{1}{4}(4\sqrt{3}-6)}{2-\sqrt{3}} \times \frac{(2+\sqrt{3})}{(2+\sqrt{3})}$$

$$= \frac{\frac{1}{4}(2\sqrt{3})}{1} = \frac{\sqrt{3}}{2}$$

So $\sin B = \frac{\sqrt{3}}{2}$ so $B = 60°$

3.



$\sin 60 = \frac{O}{H} = \frac{\sqrt{3}}{2}$ ✓

4. $DCA = 120°$ so $DAC = 15°$

so $AC \times \sin 60 = 3\sqrt{2} \times \sin 45$ [= AB]

$\frac{\sqrt{3}}{2} \times AC = 3\sqrt{2} \times \frac{1}{\sqrt{2}}$

so $AC = 2\sqrt{3}$. [cos rule also works]

$BC = AC \times \cos 60 = 2\sqrt{3} \times \frac{1}{2} = \sqrt{3}$.

In ABD using Pythagoras:

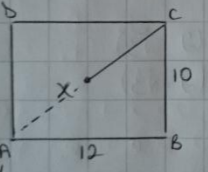
$(3\sqrt{2})^2 = (x+\sqrt{3})^2 + 3^2$

$18 = x^2 + 2\sqrt{3}x + 3 + 9$

$x^2 + 2\sqrt{3}x - 6 = 0$

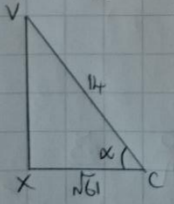
$x = \frac{-2\sqrt{3} \pm \sqrt{(12+24)}}{2} = -\sqrt{3}+3$ [$x > 0$]

5.



$CX = \sqrt{10^2+12^2} \div 2$

$= \sqrt{61}$

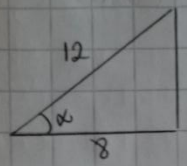


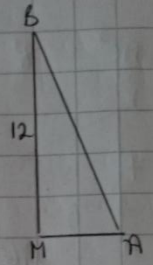
$VX = \sqrt{14^2 - (\sqrt{61})^2}$

$= \sqrt{135} = 3\sqrt{15} = 11.6\text{cm}$

$\cos\alpha = \frac{\sqrt{61}}{14}$ $\alpha = 56.1°$

6. Let M = midpoint of AC





$\cos\alpha = 8/12 \Rightarrow \alpha = 48.2°$

7. $\frac{1}{2}ab\sin C = \text{Area}$

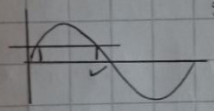
$\frac{1}{2} \times 10 \times 18 \times \sin P = 27$

$\sin P = 0.3$

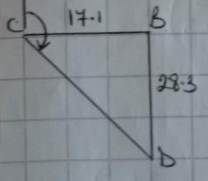
$P = \sin^{-1}(0.3) = 17.5°$ but P is obtuse

so $P = 180 - 17.5$

$= 162.5°$

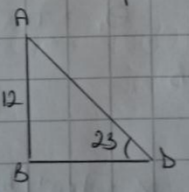


8. [3rd Step]



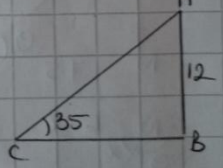
$CD = 33.0\text{m}$

[2nd Step]



$BD = \frac{12}{\tan 23} = 28.3$

[1st Step]

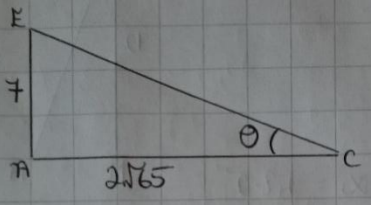


$CB = \frac{12}{\tan 35} = 17.1$

Bearing of D from C $= 90 + \tan^{-1}\frac{28.3}{17.1}$

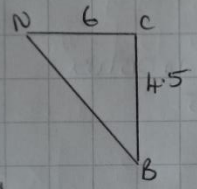
$= 149°$

9. $AC^2 = 8^2 + 14^2 \Rightarrow AC = 2\sqrt{65}$



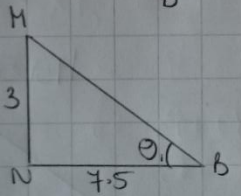
$\theta = \tan^{-1}\frac{7}{2\sqrt{65}} = 23.5°$

10.

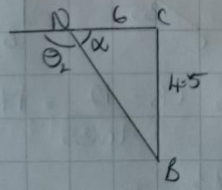


$BN = \sqrt{(4.5^2 + 6^2)}$

$= 7.5.$



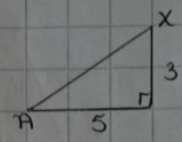
$\theta_1 = \tan^{-1}\left(\frac{3}{7.5}\right) =$

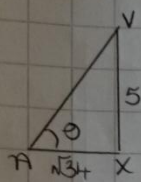


$\alpha = \tan^{-1}\left(\frac{4.5}{6}\right) = 36.9°$

so $\theta_2 = 180° - 36.9° = 143°$

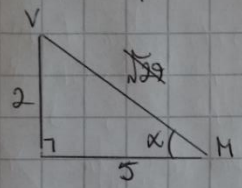
11. $(AX)^2 = 5^2 + 3^2$ $AX = \sqrt{34}$.





$\theta = \tan^{-1}\left(\frac{5}{\sqrt{34}}\right) = 40.6°$

let M = midpoint of RQ



$\alpha = \tan^{-1}\left(\frac{2}{5}\right) = 21.8°$