

Further Maths GCSE

Trigonometric Equations and Identities Answers

1. $\tan^2 x - 1 = \frac{\sin^2 x}{\cos^2 x} - 1$
 $= \frac{\sin^2 x - \cos^2 x}{\cos^2 x}$

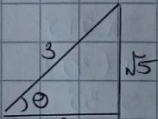
[but $\sin^2 x + \cos^2 x = 1$
 so $\sin^2 x = 1 - \cos^2 x$]

$$= \frac{(1 - \cos^2 x) - \cos^2 x}{\cos^2 x}$$

$$= \frac{1 - 2\cos^2 x}{\cos^2 x} \quad \text{QED}$$

Proved

2. $\sin \theta = \frac{o}{h} = \frac{\sqrt{5}}{3}$

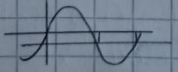


Using Pythagoras $a^2 = 3^2 - (\sqrt{5})^2$
 $a = \pm 2$

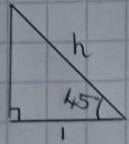
so $\cos \theta = \frac{a}{h} = \frac{-2}{3}$ (since θ is obtuse)

Alternative $\sin^2 \theta = \frac{5}{9}$ $\cos^2 \theta = 1 - \sin^2 \theta$
 $= \frac{4}{9}$
 $\cos \theta = \pm \frac{2}{3}$ since θ is obtuse

3. $(2s+1)(s-1) = 2s^2 - s - 1$
 $2\sin^2 \theta - \sin \theta - 1 = 0 \Rightarrow (2\sin \theta + 1)(\sin \theta - 1) = 0$
 so $\sin \theta = -\frac{1}{2}$ or $\sin \theta = 1$
 $\Rightarrow \theta = -30, 210, 330$ $\theta = 90^\circ$

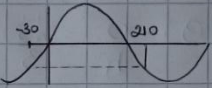


so $\theta = 90^\circ, 210^\circ, 330^\circ$

4.  Using Pythagoras $h = \sqrt{(1^2 + 1^2)} = \sqrt{2}$
 so $\sin 45 = \frac{o}{h} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

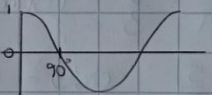
5. $5\sin^2 x - 2\sin x = 0$
 $\sin x(5\sin x - 2) = 0$
 so $\sin x = 0$ $\sin x = \frac{2}{5}$
 $x = 0, 180, 360$ $x = 23.6^\circ, 156.4^\circ$

6. $f(210^\circ) = \sin 210^\circ$
 $= \sin(-30^\circ)$
 $= -\sin 30^\circ$
 $= -\frac{1}{2}$



$-1 \leq f(x) \leq 1$

$g(x) = \cos x$



$\theta = 90^\circ$ to keep range positive.

7. $1 - \tan \theta \sin \theta \cos \theta$
 $= 1 - \frac{\sin \theta \sin \theta \cos \theta}{\cos \theta}$
 $= 1 - \sin^2 \theta$ but $\sin^2 \theta + \cos^2 \theta = 1$
 $= \cos^2 \theta$

$$8. \quad 3\cos^2\theta - 1 = 0$$

$$\cos^2\theta = \frac{1}{3}$$

$$\cos\theta = \pm\frac{1}{\sqrt{3}}$$

$$\cos\theta = \frac{1}{\sqrt{3}} \quad \text{or} \quad \cos\theta = -\frac{1}{\sqrt{3}}$$

$$\theta = 54.7^\circ \quad \text{or} \quad \theta = 125.3^\circ$$

$$9. \quad \frac{\tan\theta + 1}{\tan\theta} \equiv \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$\equiv \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$\equiv \frac{1}{\sin\theta\cos\theta} \quad \text{Proved}$$

$$10. \quad \tan^2\theta + 3\tan\theta = 0$$

$$\tan\theta(\tan\theta + 3) = 0$$

$$\tan\theta = 0 \quad \text{or} \quad \tan\theta = -3$$

$$\theta = 0^\circ, 360^\circ, 180^\circ \quad \theta = \cancel{4.6}, 108.4, 288.4$$

$$\text{so } \theta = 0, 180, 360, 108.4, 288.4$$

$\begin{matrix} \curvearrowright & \curvearrowright \\ +180 & +180 \end{matrix}$

