EQUATION, IDENTITY, EXPRESSION, FORMULA

EQUATION =

TRUE FOR SOME VALUES

$5(x+3)=3x+23$

$5x+15=3x+23$
$5x-3x=23-15$
$2x=8$
$x=4$

SOLVE

IDENTITY ≡

TRUE FOR ALL VALUES

$5(x+3)-2(x-4)\equiv 3x+23$

PROVE

$5(x+3)-2(x-4)$
$\equiv 5x+15-2x+8$
$\equiv 5x-2x+15+8$
$\equiv 3x+23$

EXPRESSION

NO EQUALS SIGN

x^2+3x+5

WHEN $x=7$
x^2+3x+5
$=49+21+5$
$=75$

SUBSTITUTE

FORMULA

A RULE LINKING TWO OR MORE VARIABLES

$V=\frac{1}{3}\pi r^2h$

REARRANGE

$3V=\pi r^2h$
$\frac{3V}{\pi r^2}=h$

USING FORMULAE

SUBSTITUTE

$a=20$
$b=-2$
$c=6$

$y=\frac{5a+bc^2}{4}$

$y=\frac{5(20)+(-2)(6)^2}{4}$
$=\frac{100+-2\times 36}{4}$
$=\frac{100-72}{4}$
$=\frac{28}{4}=7$

REARRANGE

MAKE x THE SUBJECT

$p=\sqrt{\frac{x+y}{3}}$

SQUARE BOTH SIDES

$p^2=\frac{x+y}{3}$

×3 ON BOTH SIDES

$3p^2=x+y$

−y FROM BOTH SIDES

$3p^2-y=x$

GET x ON ITS OWN

NOTATION

USE	IN PLACE OF...
ab	$a\times b$
$3y$	$y+y+y$
a^3	$a\times a\times a$
x^2y	$x\times x\times y$
$\frac{a}{b}$	$a\div b$

FUNCTIONS

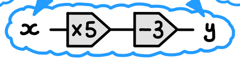
$y=5x-3$

WHEN $x=10$
$y=5(10)-3$
$y=50-3$
$y=47$

WHEN $y=32$
$32=5x-3$
$32+3=5x$
$35=5x$
$x=7$

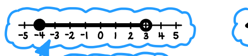
INPUT

OUTPUT

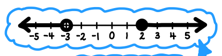


INEQUALITIES

DISPLAY ON A NUMBER LINE

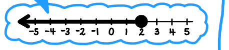


$-4\leqslant x<3$

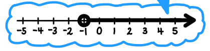


$x<-3, x>2$

$x\leqslant 2$



$x>-1$



< LESS THAN

⩽ LESS THAN OR EQUAL

> GREATER THAN

⩾ GREATER THAN OR EQUAL

SOLVING LINEAR INEQUALITIES

COLLECT x's ON THE LEFT

$3x-8<7x+12$

COLLECT x's ON THE RIGHT

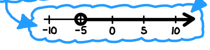
$3x-7x<12+8$
$-4x<20$
$x>\frac{20}{-4}$
$x>-5$

x TERM NEGATIVE

FLIP THE INEQUALITY

$-8-12<7x-3x$
$-20<4x$
$\frac{-20}{4}<x$
$-5<x$

x TERM POSITIVE



LINEAR EQUATIONS

x ON BOTH SIDES

−7 FROM BOTH SIDES

$2x+7=52-3x$

+3x TO BOTH SIDES

$2x+3x=52-7$
$5x=45$
$x=\frac{45}{5}$
$x=9$

COLLECT x's ON ONE SIDE

÷5 ON BOTH SIDES

WITH FRACTIONS

$\frac{3x-2}{4}-\frac{2x+5}{3}=\frac{1-x}{6}$

MULTIPLY BOTH SIDES BY 12

THE LOWEST COMMON MULTIPLE OF 3,4,6

$\frac{12(3x-2)}{4}-\frac{12(2x+5)}{3}=\frac{12(1-x)}{6}$
$3(3x-2)-4(2x+5)=2(1-x)$
$9x-6-8x-20=2-2x$
$9x-8x+2x=2+6+20$
$3x=28$
$x=\frac{28}{3}$
$x=9\frac{1}{3}$

EXPAND THE BRACKETS

COLLECT x's ON ONE SIDE

÷3 ON BOTH SIDES

WITH BRACKETS

$3(2x-5)=20-x$

EXPAND THE BRACKET

$6x-15=20-x$
$6x+x=20+15$
$7x=35$
$x=\frac{35}{7}$
$x=5$

COLLECT x's ON ONE SIDE

÷7 ON BOTH SIDES

x ON ONE SIDE

$4x-7=16$
$4x=16+7$
$4x=23$
$x=\frac{23}{4}$
$x=5\frac{3}{4}$

+7 TO BOTH SIDES

÷4 ON BOTH SIDES

SIMULTANEOUS EQUATIONS

SOLVE SIMULTANEOUSLY
$3x+2y=10$
$5x-3y=42$

SUBSTITUTION METHOD

$3x+2y=10$ ①
$5x-3y=42$ ②

REARRANGE ① TO MAKE y THE SUBJECT

① ⇒ $2y=10-3x$
$y=\frac{1}{2}(10-3x)$

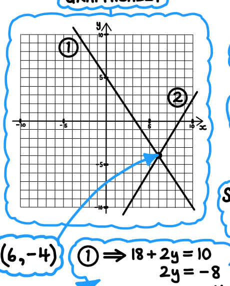
SUBSTITUTE INTO ②

$5x-3(\frac{1}{2}(10-3x))=42$

SOLVE FOR x

$5x-15+\frac{9}{2}x=42$
$10x-30+9x=84$
$19x=114$
$x=\frac{114}{19}$
$x=6$

GRAPHICALLY



$(6,-4)$

ELIMINATION METHOD

$3x+2y=10$ ①
$5x-3y=42$ ②

MULTIPLY THROUGH THE WHOLE EQUATION

$9x+6y=30$ ①×3
+ $10x-6y=84$ ②×2

ADD THE EQUATIONS TO ELIMINATE y

$19x=114$
$x=\frac{114}{19}$
$x=6$

SOLVE FOR x

① ⇒ $18+2y=10$
$2y=-8$
$y=-4$

SUBSTITUTE BACK TO FIND y

BRACKETS

SINGLE BRACKET

$3(2x+5)$

EXPAND

$3\times 2x+3\times 5$

FACTORISE

$6x+15$

$2xy(3x^2-y)$
$2xy\times 3x^2-2xy\times y$
$6x^3y-2xy^2$

$x(3x+7)$
$3x\times x+7\times x$
$3x^2+7x$

DOUBLE BRACKET

PERFECT SQUARE

$(x+5)^2$
$=(x+5)(x+5)$
$x^2+5x+5x+25$
$x^2+10x+25$

$(x+5)(x-2)$

EXPAND

$x^2+5x-2x-10$

FACTORISE

$x^2+3x-10$

DIFFERENCE OF TWO SQUARES

$(x+3)(x-3)$
$x^2+3x-3x-9$
x^2-9

SEQUENCES

GEOMETRIC SEQUENCES

TERM $\xrightarrow{\times \text{ SAME AMOUNT}}$ TERM

1ST TERM	2ND TERM	3RD TERM	4TH TERM	5TH TERM
2	6	18	54	162

×3 ×3 ×3 ×3

FIBONACCI SEQUENCES

ADD THE PREVIOUS TWO TERMS TO GET THE NEXT ONE

① ① ② ③ ⑤ ⑧ ⑬ ㉑

QUADRATIC SEQUENCES

④ ⑦ ⑫ ⑲ ㉘
+3 +5 +7 +9
+2 +2 +2

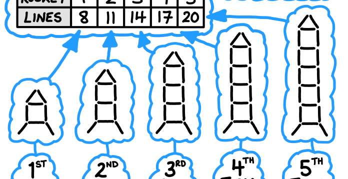
STEPS CHANGE BY THE SAME AMOUNT

ARITHMETIC SEQUENCES

TERM $\xrightarrow{+ \text{ SAME AMOUNT}}$ TERM

FROM A PATTERN

ROCKET	1	2	3	4	5
LINES	8	11	14	17	20



1ST TERM	2ND TERM	3RD TERM	4TH TERM	5TH TERM
8	11	14	17	20
3×1+5	3×2+5	3×3+5	3×4+5	3×5+5

+3 +3 +3 +3

n^{TH} TERM $=3n+5$

FIND THE 70TH TERM

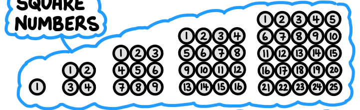
70TH TERM
$=3\times 70+5$
$=210+5$
$=215$

IS 127 IN THE SEQUENCE?

$3n+5=127$
$3n=122$
$n=\frac{122}{3}$
$n=40\frac{2}{3}$

n IS NOT AN INTEGER
⇒ 127 NOT IN THE SEQUENCE

SQUARE NUMBERS



1 4 9 16 25...

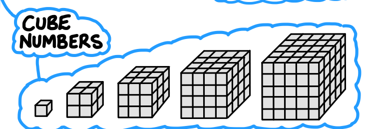
n^{TH} TERM $=n^2$

+3 +3 +3 +3 +3 +3

4 7 12 19 28...

n^{TH} TERM $=n^2+3$

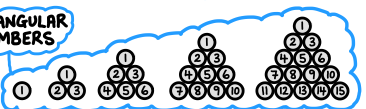
CUBE NUMBERS



1 8 27 64 125...

n^{TH} TERM $=n^3$

TRIANGULAR NUMBERS



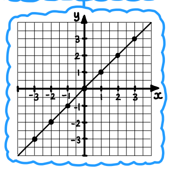
1 3 6 10 15...

n^{TH} TERM $=\frac{n(n+1)}{2}$

GRAPHS OF DIFFERENT FUNCTIONS

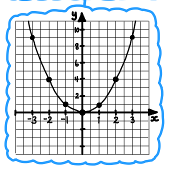
LINEAR $y=x$

x	−3	−2	−1	0	1	2	3
y	−3	−2	−1	0	1	2	3



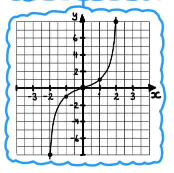
QUADRATIC $y=x^2$

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9



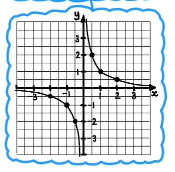
CUBIC $y=x^3$

x	−3	−2	−1	0	1	2	3
y	−27	−8	−1	0	1	8	27



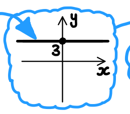
RECIPROCAL $y=\frac{1}{x}$

x	−3	−2	−1	0	1	2	3
y	$-\frac{1}{3}$	$-\frac{1}{2}$	−1	☒	1	$\frac{1}{2}$	$\frac{1}{3}$



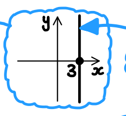
GRAPHS OF LINEAR FUNCTIONS

$y=3$



HORIZONTAL LINES

VERTICAL LINES



$x=3$

$y=mx+c$

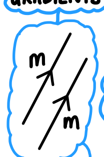
$y=\frac{2}{3}x+1$

GRADIENT

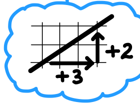
THE DIRECTION OF THE LINE

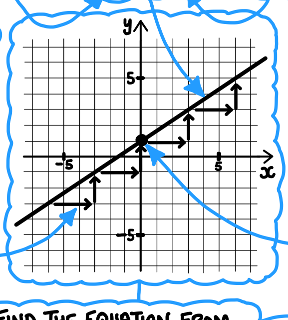
$\frac{\text{CHANGE IN } y}{\text{CHANGE IN } x}=\frac{+2}{+3}$

EQUAL GRADIENTS



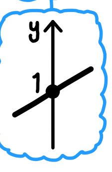
PARALLEL LINES





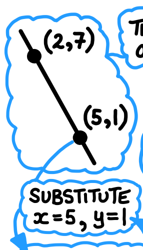
y-INTERCEPT

WHERE THE LINE CROSSES THE y-AXIS

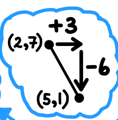


FIND THE EQUATION FROM...

TWO POINTS ON THE LINE



FIND THE GRADIENT



$m=\frac{1-7}{5-2}$
$=\frac{-6}{3}$
$=-2$

$y=-2x+c$

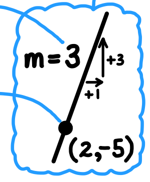
SUBSTITUTE
$x=5, y=1$

$1=-2\times 5+c$
$1=-10+c$
$1+10=c$
$11=c$

$y=-2x+11$

GRADIENT AND A POINT ON THE LINE

$y=3x+c$

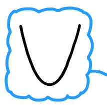


SUBSTITUTE
$x=2, y=-5$

$-5=3\times 2+c$
$-5=6+c$
$-5-6=c$
$-11=c$

$y=3x-11$

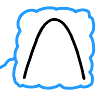
GRAPHS OF QUADRATIC FUNCTIONS



$a>0$

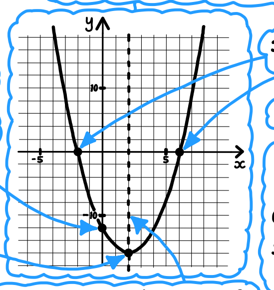
$y=ax^2+bx+c$

$a<0$



$y=x^2-4x-12$

x	−3	−2	−1	0	1	2	3	4	5	6	7
y	9	0	−7	−12	−15	−16	−15	−12	−7	0	9



y-INTERCEPT

WHEN $x=0$

$y=(0)^2-4(0)-12$
$y=-12$

TURNING POINT IS AT $(2,-16)$

LINE OF SYMMETRY IS $x=2$

x-INTERCEPTS OR ROOTS

WHEN $y=0$

SOLVE
$0=x^2-4x-12$
$0=(x-6)(x+2)$
$x-6=0, x+2=0$
$x=6, x=-2$

DISTANCE, SPEED, TIME

DISTANCE = SPEED × TIME
$=2\times 1.5$
$=3$ km

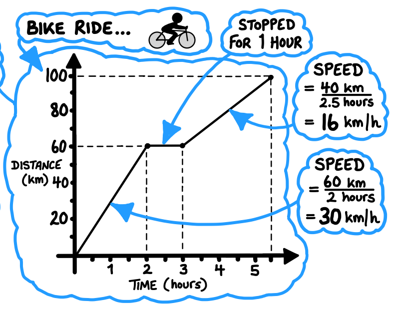
SPEED $=\frac{\text{DISTANCE}}{\text{TIME}}$
$=\frac{90}{3}=30$ km/h

e.g.	SWIM	BIKE	RUN
DISTANCE	?	90 km	21 km
SPEED	2 km/h	?	12 km/h
TIME	1h 30m	3h	?

TIME $=\frac{\text{DISTANCE}}{\text{SPEED}}$
$=\frac{21}{12}$
$=1.75$h
$=1$h 45m

TRAVEL GRAPHS

BIKE RIDE...



STOPPED FOR 1 HOUR

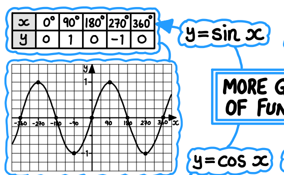
SPEED
$=\frac{40\text{ km}}{2.5\text{ hours}}$
$=16$ km/h

SPEED
$=\frac{60\text{ km}}{2\text{ hours}}$
$=30$ km/h

MORE GRAPHS OF FUNCTIONS

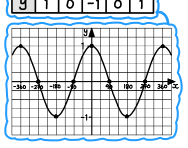
$y=\sin x$

x	0°	90°	180°	270°	360°
y	0	1	0	−1	0



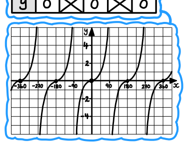
$y=\cos x$

x	0°	90°	180°	270°	360°
y	1	0	−1	0	1



$y=\tan x$

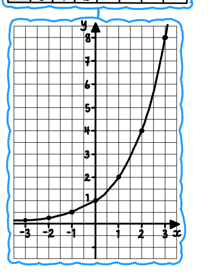
x	0°	90°	180°	270°	360°
y	0	☒	0	☒	0



$y=2^x$

COULD BE ANY POSITIVE INTEGER

x	−3	−2	−1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



ALGEBRAIC FRACTIONS

×

SIMPLIFY $\frac{5x-10}{3xy}\times\frac{x}{x-2}$

$=\frac{x(5x-10)}{3xy(x-2)}$

MULTIPLY NUMERATORS, MULTIPLY DENOMINATORS

$=\frac{5\cancel{x}\cancel{(x-2)}}{3\cancel{x}y\cancel{(x-2)}}$

$=\frac{5}{3y}$

CANCEL

CANCEL

SIMPLIFY
$\frac{x^2-16}{3x^2+10x-8}$

FACTORISE

$=\frac{(x+4)(x-4)}{(3x-2)(x+4)}$

$=\frac{x-4}{3x-2}$

CANCEL

+ AND −

SIMPLIFY $\frac{5}{x-3}-\frac{4}{x+2}$

$\times\frac{(x+2)}{(x+2)}$

$\times\frac{(x-3)}{(x-3)}$

$=\frac{5(x+2)}{(x-3)(x+2)}-\frac{4(x-3)}{(x+2)(x-3)}$

EXPAND BRACKETS

$=\frac{(5x+10)-(4x-12)}{(x-3)(x+2)}$

COMMON DENOMINATOR

$=\frac{5x+10-4x+12}{(x-3)(x+2)}$

COLLECT LIKE TERMS

$=\frac{x+22}{(x-3)(x+2)}$

÷

SIMPLIFY
$\frac{5ab}{12}\div\frac{2a}{9b^2}$

FLIP THE SECOND FRACTION AND ×

$=\frac{5ab}{12}\times\frac{9b^2}{2a}$

$=\frac{45ab^3}{24a}$

FACTORISE

$=\frac{3\cancel{a}\times 15b^3}{3\cancel{a}\times 8}$

CANCEL

$=\frac{15b^3}{8}$

MORE BRACKETS

EXPAND THREE BRACKETS

$(3x+1)(x+4)(2x-5)$

EXPAND THE FIRST TWO BRACKETS

$=(3x^2+12x+x+4)(2x-5)$
$=(3x^2+13x+4)(2x-5)$

EXPAND AGAIN

×2x

×−5

$=6x^3+26x^2+8x$
$\quad -15x^2-65x-20$

COLLECT LIKE TERMS

$6x^3+11x^2-57x-20$

FACTORISE
ax^2+bx+c

$2x^2\,\boxed{+5x}\,-12$

FIND ac

$2\times -12=\boxed{-24}$

$\boxed{+8}\ \boxed{-3}$

FACTORS OF −24 WHICH SUM TO +5

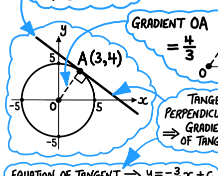
SPLIT THE MIDDLE TERM

$=2x^2\,\boxed{+8x}\,\boxed{-3x}\,-12$
$=2x^2+8x\ -3x-12$
$=2x(x+4)\ -3(x+4)$
$=(2x-3)(x+4)$

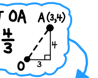
FACTOR OUT $(x+4)$

EQUATION OF A CIRCLE

FIND THE EQUATION OF THE TANGENT TO THE CIRCLE AT A GIVEN POINT



GRADIENT OA
$=\frac{4}{3}$



TANGENT IS PERPENDICULAR TO OA
⇒ GRADIENT OF TANGENT $=-\frac{3}{4}$

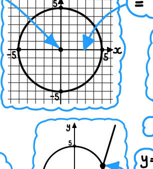
EQUATION OF TANGENT ⇒ $y=-\frac{3}{4}x+c$
TANGENT PASSES THROUGH (3,4)
⇒ $4=-\frac{3}{4}(3)+c$ ⇒ $c=\frac{25}{4}$

EQUATION OF THE TANGENT IS
$y=-\frac{3}{4}x+\frac{25}{4}$

CENTRE
$(0,0)$

$x^2+y^2=25$

RADIUS
$=\sqrt{25}$
$=5$



SOLVE LINEAR AND QUADRATIC SIMULTANEOUSLY

$x^2+y^2=25$
$y=3x-13$

SUBSTITUTE LINEAR INTO QUADRATIC

$x^2+(3x-13)^2=25$
$x^2+9x^2-78x+169=25$
$10x^2-78x+144=0$
$5x^2-39x+72=0$

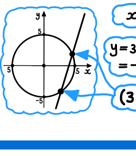
EXPAND AND SIMPLIFY

SOLVE THE QUADRATIC EQUATION

$x=3$, $x=4.8$

$y=3(3)-13$, $y=3(4.8)-13$
$=-4$ $=1.4$

SUBSTITUTE BACK INTO LINEAR



$(3,-4)$ AND $(4.8,1.4)$

MORE QUADRATICS

SOLVING EQUATIONS

$2x^2+3x-7=0$

$ax^2+bx+c=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

THE QUADRATIC FORMULA

$x=\frac{-3\pm\sqrt{(3)^2-4(2)(-7)}}{2(2)}$
$x=\frac{-3\pm\sqrt{9--56}}{4}$
$x=\frac{-3\pm\sqrt{65}}{4}$
$x=1.27, x=-2.77$

REARRANGE TO FIND THE QUADRATIC

$\frac{x}{2x+5}=\frac{2}{x+1}$

$x(x+1)=2(2x+5)$
$x^2+x=4x+10$
$x^2-3x-10=0$
$(x-5)(x+2)=0$
$x=5, x=-2$

COMPLETING THE SQUARE

$x^2-6x+2=0$

$(x-3)^2-9+2=0$
$(x-3)^2-7=0$
$(x-3)^2=7$
$x-3=\pm\sqrt{7}$
$x=3\pm\sqrt{7}$
$x=3-\sqrt{7}, x=3+\sqrt{7}$

SQUARE ROOT BOTH SIDES

$\boxed{x^2+8x}+21$

HALVE THE +8

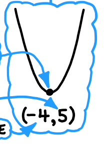
SUBTRACT $(+4)^2$

$\boxed{(x+4)^2-16}+21$

$(x+4)^2+5$

$y=x^2-8x+21$

FIND THE TURNING POINT



y-VALUE

NEGATIVE x-VALUE

$(-4,5)$

n^{TH} TERM FOR QUADRATIC SEQUENCES

n^{TH} TERM $=an^2+bn+c$

④ ⑬ ㉖ ㊸ (64)...
+9 +13 +17 +21
+4 +4 +4

CONSTANT SECOND DIFFERENCE $=2a$

$2a=4$ ⇒ $a=2$

1^{ST} TERM $=2(1)^2+b(1)+c=4$
2^{ND} TERM $=2(2)^2+b(2)+c=13$

$2+b+c=4$
$8+2b+c=13$

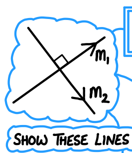
$b+c=2$
$2b+c=5$

$b=3$
$c=-1$

SOLVE SIMULTANEOUSLY TO FIND b AND c

n^{TH} TERM $=2n^2+3n-1$

PERPENDICULAR LINES



$m_1m_2=-1$
OR $m_1=\frac{-1}{m_2}$

SHOW THESE LINES ARE PERPENDICULAR

$y=-\frac{2}{5}x+1$

$5x-2y=14$

$5x-14=2y$
$y=\frac{5}{2}x-7$

$m_1=-\frac{2}{5}$

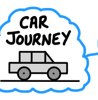
$m_2=\frac{5}{2}$

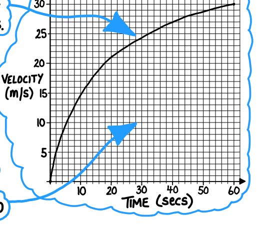
$m_1m_2=-\frac{2}{5}\times\frac{5}{2}=-1$
SO THE LINES ARE PERPENDICULAR

ESTIMATING AREA AND GRADIENT

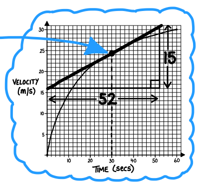
ESTIMATE THE ACCELERATION AFTER 30 secs.

CAR JOURNEY





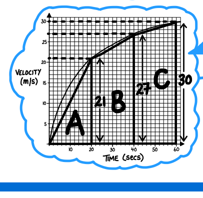
FIND THE GRADIENT OF THE TANGENT TO THE CURVE



ACCELERATION AT 30 secs.
$=\frac{15\text{ m/s}}{52\text{ s}}$
$=0.288$ m/s²

ESTIMATE THE DISTANCE TRAVELLED IN 60 secs.

AREA UNDER THE CURVE BETWEEN TIME 0 AND 60



$A=\frac{1}{2}\times 21\times 20\ =210$
+
$B=\frac{1}{2}\times(21+27)\times 20=480$
+
$C=\frac{1}{2}\times(27+30)\times 20=570$
1260m

AREA OF TRIANGLE

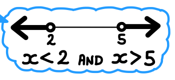
AREA OF TRAPEZIUM

DISTANCE TRAVELLED

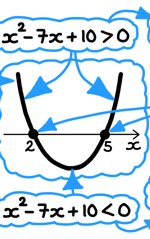
MORE INEQUALITIES

SKETCH THE GRAPH OF THE QUADRATIC FUNCTION

$x^2-7x+10>0$



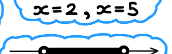
$x<2$ AND $x>5$



$x^2-7x+10=0$
$(x-2)(x-5)=0$
$x=2, x=5$

SOLVE QUADRATIC INEQUALITIES

$x^2-7x+10<0$



$2<x<5$

SOLVE INEQUALITIES IN TWO VARIABLES

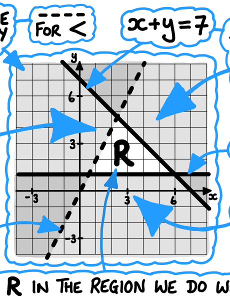
LABEL THE REGION R WHERE ALL THREE INEQUALITIES ARE SATISFIED
$y\geqslant 1, y<2x, x+y\leqslant 7$

DRAW THE BOUNDARY LINES

- - - - FOR <

——— FOR ⩽

$x+y=7$



$x+y\leqslant 7$
SO DON'T WANT THIS...

$y<2x$
SO DON'T WANT THIS...

$y=1$

$y\geqslant 1$
SO DON'T WANT THIS...

$y=2x$

WRITE R IN THE REGION WE DO WANT

FUNCTION NOTATION

COMPOSITE FUNCTION

$fg(x)=f(g(x))=f\left(\frac{x^2+5}{4}\right)=8\left(\frac{x^2+5}{4}\right)-3$
$=2(x^2+5)-3$
$=2x^2+10-3$
$=2x^2+7$

$f(3x+1)=8(3x+1)-3$
$=24x+8-3$
$=24x+5$

$g(x)=\frac{x^2+5}{4}$

EXPRESSION

FUNCTION

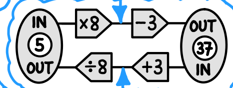
NUMBER

INPUT A ...

$f(5)=8(5)-3$
$=40-3$
$=37$

$f(x)=8x-3$

MAKE x THE SUBJECT



$y=8x-3$
$y+3=8x$
$\frac{y+3}{8}=x$

$f^{-1}(37)=\frac{37+3}{8}$
$=\frac{40}{8}$
$=5$

$f^{-1}(x)=\frac{x+3}{8}$

WRITE AS THE INVERSE FUNCTION IN TERMS OF x

INVERSE FUNCTION

PROOF

PROVE THAT
$(2n+1)^2-(2n+1)$ IS AN EVEN NUMBER
FOR ALL POSITIVE INTEGER VALUES OF n

$(2n+1)^2-(2n+1)$
$=(2n+1)(2n+1)-(2n+1)$
$=4n^2+4n\cancel{+1}-2n\cancel{-1}$
$=4n^2+2n$
$=2(2n^2+n)$

SINCE n IS A POSITIVE INTEGER
$(2n^2+n)$ IS ALSO A POSITIVE INTEGER
$2(2n^2+n)=2$(POSITIVE INTEGER)
$2(2n^2+n)$ IS AN EVEN NUMBER
⇒ $(2n+1)^2-(2n+1)$ IS AN EVEN NUMBER

ITERATION

FIND AN APPROXIMATE SOLUTION TO THE EQUATION

$2x^2-x^3+7=0$

REARRANGE TO GET x ON ITS OWN

$x^3=2x^2+7$
$x=\frac{2x^2+7}{x^2}$
$x=2+\frac{7}{x^2}$

CONVERT TO AN ITERATIVE FORMULA

$x_{n+1}=2+\frac{7}{x_n^2}$

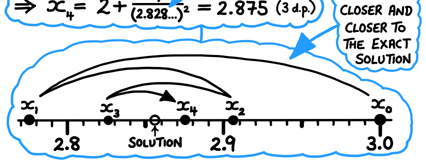
INITIAL VALUE ⇒ $x_0=3$

CHOOSE x_0 CLOSE TO THE SOLUTION

⇒ $x_1=2+\frac{7}{(3)^2}=2.778$ (3 d.p.)
⇒ $x_2=2+\frac{7}{(2.777...)^2}=2.907$ (3 d.p.)
⇒ $x_3=2+\frac{7}{(2.907...)^2}=2.828$ (3 d.p.)
⇒ $x_4=2+\frac{7}{(2.828...)^2}=2.875$ (3 d.p.)

ROUND ANSWERS BUT SUBSTITUTE EXACT VALUES BACK IN

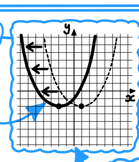
VALUES GET CLOSER AND CLOSER TO THE EXACT SOLUTION

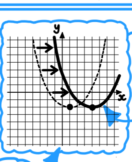


TRANSFORMATIONS OF GRAPHS

$y=f(x+3)$

TRANSLATE 3 UNITS TO THE LEFT

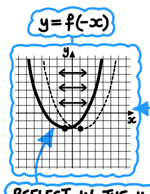




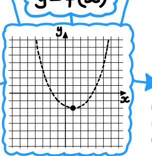
$y=f(x-3)$

TRANSLATE 3 UNITS TO THE RIGHT

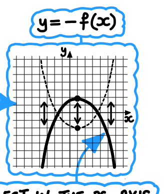
$y=f(-x)$



$y=f(x)$



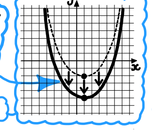
$y=-f(x)$



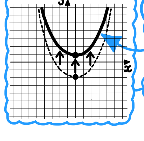
REFLECT IN THE y-AXIS

REFLECT IN THE x-AXIS

TRANSLATE DOWN BY 3 UNITS



$y=f(x)-3$



TRANSLATE UP BY 3 UNITS

$y=f(x)+3$