ARITHMETIC

73.65 − 28.19 = 45.46

2819 − 4546

2617 + 945 = 3562

26.17 + 9.45 = 35.62

43.05 ÷ 1.5 = 4305/100 ÷ 15/10 = 4305/100 × 10/15 = 4305/15 × 10/100 = 287 × 1/10 = 28.7

15 | 4 3 0 5 = 0287

259 × 37 = 9583

2.59 × 370 = (259/100) × (37×10) = 259 × 37 × 10/100 = 9583 × 1/10 = 958.3

NEGATIVE NUMBERS

7×−5=−35 −7×−5=35 −20÷5=−4 −20÷−5=4

× AND ÷

+×+=+ +×−=− −×+=− −×−=+

+÷+=+ +÷−=− −÷+=− −÷−=+

+ AND −

−2−+5=−2−5=−7 −2+−5=−2−5=−7

−2++5=−2+5=3 −2−−5=−2+5=3

−5 +5

−8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4

SUBTRACTING THE NEGATIVE AMOUNT = ADDING THE POSITIVE AMOUNT

APPROXIMATION

3784 × 72 ≈ 4000 × 70 = 280000

ESTIMATING

ROUND VALUES TO 1 SIGNIFICANT FIGURE

APPROX. EQUAL TO

8792 − 958 / 21.6 ≈ 9000 − 1000 / 20 = 8000/20 = 400

ROUNDING

0.0782904 → (Round To 3 d.p.) → 0.078

0.078 0.078②904 0.079 — Round Down

723608 → (Round To 3 s.f.) → 724000

723000 723⑥08 724000 — Round Up

DECIMAL PLACES

62.954871 → (Round To 1 d.p.) → 63.0

62.9 62.9⑤4871 63.0 — Round Up

SIGNIFICANT FIGURES

0.003748 → (Round To 2 s.f.) → 0.0037

0.0037 0.0037④8 0.0038 — Round Down

THE NEXT DIGIT IS... 9 8 7 6 5 — ROUND UP; 4 3 2 1 0 — ROUND DOWN

FRACTIONS

2 3/4 × 1 2/5 = 11/4 × 7/5 = 77/20 = 3 17/20

CONVERT TO IMPROPER FRACTIONS

CONVERT BACK TO A MIXED NUMBER

OPERATIONS WITH MIXED NUMBERS

11/4 — 11 ÷ 4 = 2 r 3 — 11 = 2×4+3 — 2 3/4

IMPROPER FRACTION ↔ MIXED NUMBER

FRACTION OF A QUANTITY

FIND 3/5 OF £60

£60 ÷ 5 = £12

£12 × 3 = £36

£12 £12 £12 £12 £12

CANCELLED TO ITS LOWEST TERMS

60/90 ÷2 30/45 ÷3 10/15 ÷5 2/3

60/90 ÷2 30/45 ÷3 10/15 ÷5 2/3

EQUIVALENT FRACTIONS

FOUR OPERATIONS

2/3 × 4/5 = 2×4 / 3×5 = 8/15

MULTIPLY NUMERATORS, MULTIPLY DENOMINATORS

3/7 ÷ 5/8 = 3/7 × 8/5 = 3×8 / 7×5 = 24/35

FLIP THE SECOND FRACTION AND MULTIPLY

2/5 + 3/7 = 2×7/5×7 + 3×5/7×5 = 14/35 + 15/35 = 29/35

FIND A COMMON DENOMINATOR

LCM OF 5 AND 7 IS 35

LCM OF 6 AND 8 IS 24

USE EQUIVALENT FRACTIONS WITH A COMMON DENOMINATOR

5/6 − 3/8 = 5×4/6×4 − 3×3/8×3 = 20/24 − 9/24 = 11/24

0.75 = 3/4

3.5 ≠ 3 1/5

COMPARING NUMBERS

−5 < −3

1/3 > 1/5

0.28 = 28/100 = 7/20

4 ÷ 5 = 0.8

0.8 × 100 = 80%

FRACTION	DECIMAL	PERCENT
4/5	?	?
?	0.28	?
?	?	62.5%

0.28 × 100 = 28%

0.625 = 625/1000 = 125/200 = 25/40 = 5/8

62.5 ÷ 100 = 0.625

FRACTION, DECIMAL, PERCENTAGE

FRACTION → (WRITE AS 1/10, 1/100, 1/1000, ... AND CANCEL TO LOWEST TERMS)

FRACTION → (NUMERATOR ÷ DENOMINATOR) → DECIMAL

DECIMAL → (×100) → PERCENTAGE

DECIMAL → (÷100) → FRACTION

PERCENTAGE → (×100) → DECIMAL

FACTORS MULTIPLES AND PRIMES

1 IS NOT A PRIME

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ...

HAVE EXACTLY TWO FACTORS, THE NUMBER ITSELF AND 1

PRIME NUMBERS

LOWEST COMMON MULTIPLE OF 12 AND 30 IS 60

12, 24, 36, 48, 60, ... — MULTIPLES OF 12

30, 60, 90, 120, 150, ... — MULTIPLES OF 30

MULTIPLES

NUMBERS IN THAT "TIMES TABLE"

FACTORS

INTEGERS WHICH DIVIDE EXACTLY INTO THE NUMBER WITH NO REMAINDER

FACTORS OF 30: 1, 2, 3, 5, 6, 10, 15, 30

FACTORS OF 12: 1, 2, 3, 4, 6, 12

HIGHEST COMMON FACTOR OF 12 AND 30 IS 6

PRIME FACTORISATION

EXPRESSING AN INTEGER AS A PRODUCT OF PRIME NUMBERS

THIS EXPRESSION IS UNIQUE TO ANY INTEGER

132 = 12 × 11 = 6 × 2 × 11 = 2 × 3 × 2 × 11 = 2² × 3 × 11

420 = 42 × 10 = 6 × 7 × 2 × 5 = 2 × 3 × 2 × 5 × 7 = 2² × 3 × 5 × 7

HIGHEST COMMON FACTOR = 2 × 2 × 3 = 12

LOWEST COMMON MULTIPLE = 2 × 2 × 3 × 5 × 7 × 11 = 4620

UNITS OF MEASURE

VOLUME = 642 cm³

MASS = 12400 g

GOLD

DENSITY OF GOLD = 12400/642 = 19.3 g/cm³

M D V

UNITS OF MASS / UNITS OF VOLUME

COMPOUND MEASURES

EXCHANGE RATES

$797.34 ÷1.37 £582.00

×1.37 ×1.37

£1 = $1.37

METRIC ↔ IMPERIAL

30.48 cm ÷2.54 12 inches

×2.54

1 inch = 2.54 cm

cm ÷2.54 inch

×2.54

METRIC ↔ METRIC

LENGTH metres: mm ÷10 cm ÷100 m ÷1000 km

mm ×10 cm ×100 m ×1000 km

27800 mm (×10 ÷10) 2780 cm (×100 ÷100) 27.8 m (×1000 ÷1000) 0.0278 km

AREA square metres: mm² ÷10² cm² ÷100² m²

mm² ×10² cm² ×100² m²

1 m² = 10000 cm² (100 cm × 100 cm)

VOLUME cubic metres: mm³ ÷10³ cm³ ÷100³ m³

mm³ ×10³ cm³ ×100³ m³

1 m³ = 1000000 cm³ (100 cm × 100 cm × 100 cm)

1 ml = 1 cm³

CAPACITY litres: ml ÷1000 l

ml ×1000 l

MASS grammes: mg ÷1000 g ÷1000 kg

mg ×1000 g ×1000 kg

ORDER OF OPERATIONS

BRACKETS

INDICES

DIVISION MULTIPLICATION LEFT TO RIGHT

ADDITION SUBTRACTION LEFT TO RIGHT

18 ÷ 2 + (7−3) × 5² − 10

= 18 ÷ 2 + 4 × 5² − 10 (BRACKET)

= 18 ÷ 2 + 4 × 25 − 10 (INDICES)

= 9 + 100 − 10 (÷ AND ×)

= 99 (+ AND −)

POWERS AND ROOTS

POWER OR INDEX

BASE

2⁵ = 2×2×2×2×2 (REPEATED MULTIPLICATION)

2×2×2×2×2 = 32

2⁵ = 32

THE FIFTH ROOT: ⁵√32 = 2

THE LAWS OF INDICES (POWERS)

MULTIPLYING WITH THE SAME BASE: x^a × x^b = x^(a+b) — ADD THE POWERS

2⁴ × 2³ = (2×2×2×2) × (2×2×2) = 2⁷

DIVIDING WITH THE SAME BASE: x^a ÷ x^b = x^(a−b) — SUBTRACT THE POWERS

2⁵ ÷ 2² = 2×2×2×2×2 / 2×2 = 2×2×2 = 2³

CONSIDER 2³ ÷ 2³ = 2^(3−3) = 2⁰ = 8/8 = 1

2⁰ = 1

POWER OF ZERO: x⁰ = 1 — EQUALS ONE FOR ANY NON-ZERO BASE

CONSIDER 2² ÷ 2⁵ = 2^(2−5) = 2^(−3) = 2×2 / 2×2×2×2×2 = 1/(2×2×2) = 1/2³

2^(−3) = 1/2³

NEGATIVE POWERS: x^(−a) = 1/x^a — ONE DIVIDED BY THE BASE TO THE POSITIVE POWER

POWER TO A POWER: (x^a)^b = x^(a×b) — MULTIPLY THE POWERS

(2⁴)³ = (2×2×2×2) × (2×2×2×2) × (2×2×2×2) = 2¹²

WITH ALGEBRA

a² ÷ a⁵ = a^(2−5) = a^(−3) = 1/a³

(5ab²)³ = (5)³(a)³(b²)³ = 125a³b⁶

x⁶ × x² = x^(6+2) = x⁸

(m³)⁴ = m^(3×4) = m¹²

12s⁵t² / 3s²t = 12/3 × s⁵/s² × t²/t = 4s³t

SQUARE NUMBERS

1² = 1	√1 = 1
2² = 4	√4 = 2
3² = 9	√9 = 3
4² = 16	√16 = 4
5² = 25	√25 = 5
6² = 36	√36 = 6
7² = 49	√49 = 7
8² = 64	√64 = 8
9² = 81	√81 = 9
10² = 100	√100 = 10
11² = 121	√121 = 11
12² = 144	√144 = 12
13² = 169	√169 = 13
14² = 196	√196 = 14
15² = 225	√225 = 15

POWERS OF 2

2¹ = 2	
2² = 4	√4 = 2
2³ = 8	³√8 = 2
2⁴ = 16	⁴√16 = 2
2⁵ = 32	⁵√32 = 2

POWERS OF 3

3¹ = 3	
3² = 9	√9 = 3
3³ = 27	³√27 = 3
3⁴ = 81	⁴√81 = 3
3⁵ = 243	⁵√243 = 3

POWERS OF 4

4¹ = 4	
4² = 16	√16 = 4
4³ = 64	³√64 = 4
4⁴ = 256	⁴√256 = 4
4⁵ = 1024	⁵√1024 = 4

POWERS OF 5

5¹ = 5	
5² = 25	√25 = 5
5³ = 125	³√125 = 5
5⁴ = 625	⁴√625 = 5
5⁵ = 3125	⁵√3125 = 5

POWERS OF 10

10¹ = 10	
10² = 100	√100 = 10
10³ = 1000	³√1000 = 10
10⁴ = 10 000	⁴√10 000 = 10
10⁵ = 100 000	⁵√100 000 = 10
10⁶ = 1 000 000	⁶√1 000 000 = 10

MENU

STARTER
- (T) TOMATO SOUP
- (P) PRAWN COCKTAIL

MAIN
- (C) CHICKEN PASTA
- (F) FISH + CHIPS
- (V) VEGTABLE CURRY

DESSERT
- (I) ICE CREAM
- (L) LEMON PIE

SYSTEMATIC LISTING

CHOOSE A STARTER, A MAIN AND A DESSERT

HOW MANY DIFFERENT MENU CHOICES?

TCI TCL TFI TFL TVI TVL PCI PCL PFI PFL PVI PVL

12 POSSIBILITIES

STANDARD FORM

8.20000. (1 2 3 4 5)

8.2 × 10⁵ = 8.2 × 10 × 10 × 10 × 10 × 10 = 820 000

MUST BE BETWEEN 1 AND 10

+ FOR BIG NUMBERS

3.72 × 10^(−5)

− FOR SMALL NUMBERS

5.39 × 10^(−4) = 5.39 × 1/10⁴ = 0.000539

0.0005.39 (1 2 3 4)

RECURRING DECIMALS

LET THE RECURRING DECIMAL EQUAL x.

USE SUBTRACTION WITH 10x, 100x, 1000x, ...

TO CANCEL THE DECIMAL PART AND THEN SIMPLIFY IF POSSIBLE.

LET x = 0.3̇2̇8̇

1000x = 328.2̇8̇

− 10x = 3.2̇8̇

990x = 325

x = 325/990 = 65/198

DECIMAL: 0.583̇

FRACTION: 7/12

USE LONG DIVISION: 12 | 7.5 8 3 3 3 ... = 7 ÷ 12

SURDS

(5+√2)(4−√3) = 20 + 4√2 − 5√3 − √6

EXPAND

√(a×b) = √a × √b

SIMPLIFY: √75 = √25 × √3 = √25 × √3 = 5√3

LOOK FOR A FACTOR WHICH IS A SQUARE

RATIONALISE THE DENOMINATOR

8/(3+√5) = 8/(3+√5) × (3−√5)/(3−√5) = (24 − 8√5)/(9 + 3√5 − 3√5 − 5) = (24 − 8√5)/4 = 6 − 2√5

SWAP THE + FOR −

√5 × √5 = 5

√8/√6 = √8/√6 × √6/√6 = √48/6 = 4√3/6 = 2√3/3

SIMPLIFY

SURD REMOVED FROM THE DENOMINATOR

FRACTIONAL POWERS

LAWS OF INDICES: x^a × x^b = x^(a+b)

a^(1/2) × a^(1/2) = a^(1/2+1/2) = a¹ = a

a^(1/3) × a^(1/3) × a^(1/3) = a^(1/3+1/3+1/3) = a¹ = a

a^(1/4) × a^(1/4) × a^(1/4) × a^(1/4) = a^(1/4+1/4+1/4+1/4) = a¹ = a

a^(1/2) = √a a^(1/3) = ³√a a^(1/4) = ⁴√a

a^(1/m) = ᵐ√a

125^(1/3) = ³√125 = 5

125^(2/3) = 125^(1/3 × 2) = (125^(1/3))² = 5² = 25

LAWS OF INDICES: x^(a×b) = (x^a)^b

a^(n/m) = (ᵐ√a)^n

COUNTING METHODS

A B C D E F G H

HOW MANY WAYS CAN THE 3 MEDALS BE AWARDED TO THE 8 ATHLETES?

GOLD SILVER BRONZE

8 × 7 × 6 = 336

8 TIES, 5 SHIRTS, 6 TROUSERS

HOW MANY DIFFERENT OUTFITS?

MULTIPLY THE NUMBER OF POSSIBILITIES FOR EACH ITEM

8 × 5 × 6 = 240

UPPER AND LOWER BOUNDS

HEIGHT MEASURED TO THE NEAREST 10 cm: 1.8 m

LOWER BOUND 1.7 — UPPER BOUND 1.9

1.75 ≤ h < 1.85

HEIGHT MEASURED TO THE NEAREST 1 cm: 1.37 m

LOWER BOUND 1.36 — UPPER BOUND 1.38

1.365 ≤ h < 1.375

MAXIMUM DIFFERENCE IN HEIGHTS = 1.85 − 1.365 = 0.485 m