Further Maths GCSE
Factor Theorem Answers

1. Let $x=2$ then x^3+8x^2+x-42
$= 8+32+2-42$
$= 0$ so $(x-2)$ is a factor.

2. $2x^2-8x+9 = 2[x^2-4x+4.5]$
$= 2[(x-2)^2+0.5]$
All parts are >0 $\therefore 2x^2-8x+9>0$

3. $x^3+ax^2+bx+150 = (x+c)^2(x+d)$
$=(x^2+2xc+c^2)(x+d)$
$= x^3+(2c+d)x^2+(2dc+c^2)x+c^2d$

so $c^2d=150$ $c\neq 1$ only other square factor of 150 is 25 so $c^2=25$ so $c=5$, $d=6$.

Compare x^2 terms $a = 2c+d = 16$
x terms $b = 2dc+c^2 = 85$.

4. If $x+3$ is a factor $f(-3)=0$
$\Rightarrow -27+54-3a-12=0 \Rightarrow a=5$.
$x^3+6x^2+5x-12 = (x+3)(x^2+bx-4)$

Comparing x^2 terms $6 = b+3 \Rightarrow b=3$

$= (x+3)(x^2+3x-4)$
$= (x+3)(x+4)(x-1)$

5. $f(5)=0 \Rightarrow 125-150+5a-20=0$
so $a=9$

6. $f(a)=0 \Rightarrow 2a^3-7a^2+3a=0$
$a(2a^2-7a+3)=0$
$a(2a-1)(a-3)$
so $a = 0, \frac{1}{2}, 3$ largest value $= 3$.

7. $f(1) = 1^3-21\times1+20=0$ so $x-1$ is a factor
$f(4) = 4^3-21\times4+20=0$ so $x-4$ is a factor.

8. $f(2)=0$ $8+4a+2b+24=0$
$4a+2b=-32 \Rightarrow 2a+b=-16$.
$f(-3)=0$ $-27+9a-3b+24=0$ +
$9a-3b=3 \Rightarrow 3a-b=1$
$5a=-15$
$a=-3$
$b=-10$.

9. $f(1) = 1-10+29-20=0$ so $x-1$ is a factor
$f(4)$ $64-160+116-20=0$ $x-4$ is a factor.