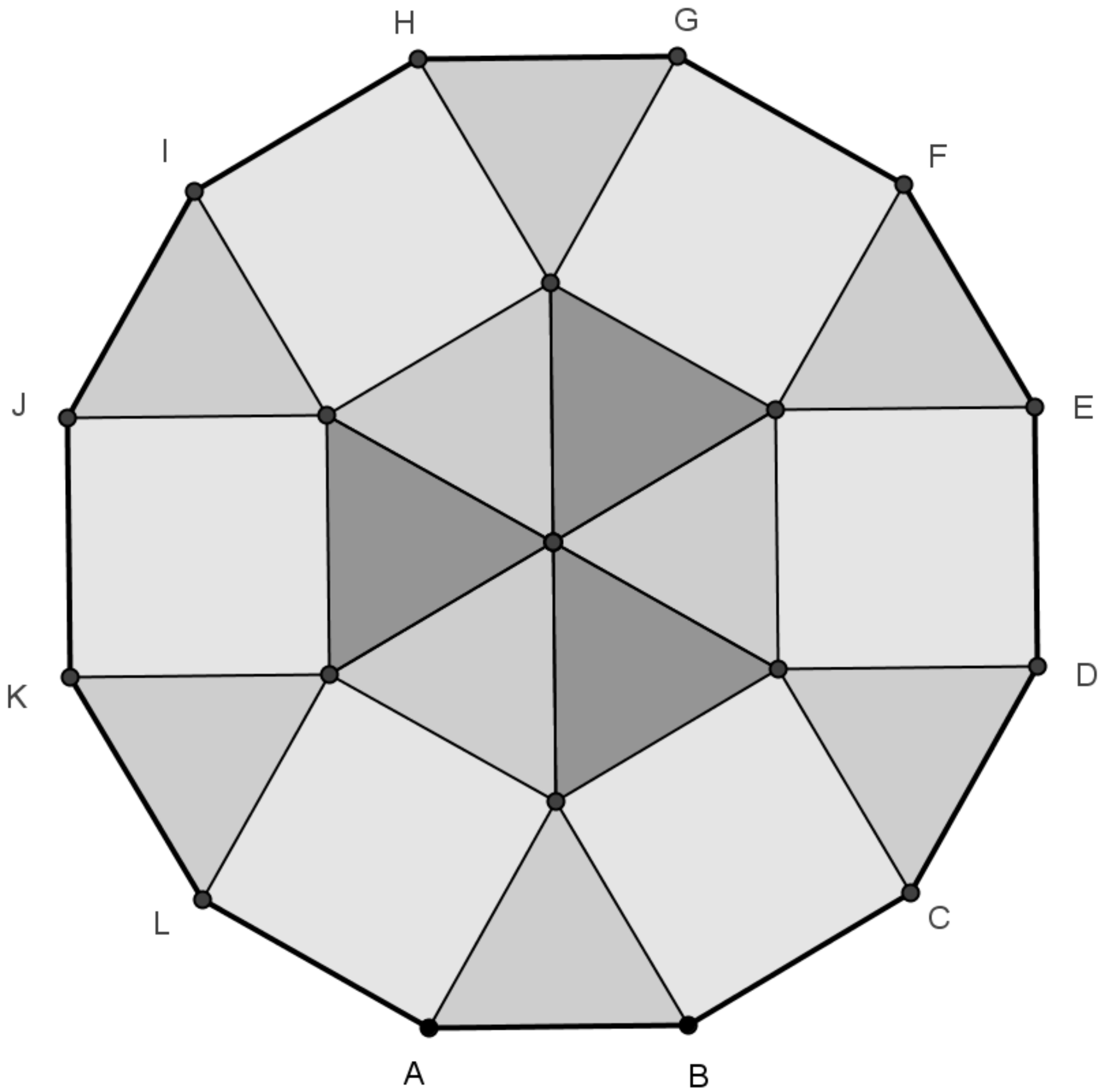
Diagonals in a Dodecagon **Extended Activity**

Student activity

In the regular dodecagon below there are 5 different lengths of diagonal.

Assuming that the side length of the dodecagon is 2, how long is each of these diagonals?

Write your answers in the simplest surd form possible.



All triangles shown are equilateral and all quadrilaterals shown are squares.

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Student help sheet

- You need to be familiar with Pythagoras' theorem.
- You will also need to know how to manipulate surds. Make sure you understand the example below and practice on the questions given if necessary.

Expanding the expression: $\left(\sqrt{a} + \sqrt{b}\right)^2 = a + b + 2\sqrt{ab}$

Hence, to find the square root of an expression involving surds, it is sometimes possible to manipulate the expression in order to obtain something in the form of: $a + b + 2\sqrt{ab}$

This can then be factorised to: $\left(\sqrt{a} + \sqrt{b}\right)^2$

Finding the square root of this gives: $(\sqrt{a} + \sqrt{b})$

Example:

Simplify $\sqrt{11 + 4\sqrt{6}}$

This has to be made '2'

$$\sqrt{11 + 4\sqrt{6}} = \sqrt{11 + 2\sqrt{24}}$$

$$= \sqrt{8 + 3 + 2\sqrt{8 \times 3}}$$

$$= \sqrt{(\sqrt{8} + \sqrt{3})^2}$$

$$= \sqrt{8} + \sqrt{3}$$

Try the method to simplify the following expressions for practice:

a) $\sqrt{15 + 10\sqrt{2}}$

b) $\sqrt{18 + 6\sqrt{5}}$

c) $\sqrt{24 + 6\sqrt{7}}$

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Teacher notes

Content:

- Manipulating surds (advanced)
- Using a realistic context from which surds naturally arise
- Using Pythagoras' theorem
- Geometrical reasoning

Possible uses:

- As a challenging task for individual students or pairs
- As a challenging task for groups or the whole class

Resource options:

- Worksheet for individual students
- Help sheet for individual students
- Student sheet with 2 dodecagons to work on
- PowerPoint file for whole class display

Possible lesson structure for a whole class lesson:

- Show PowerPoint slide 1 (or hand out sheets) and ask how many different length diagonals there are within the regular dodecagon. Students may not know that a diagonal is any straight line joining 2 non-adjacent vertices.
- Show slide 2. Tell students that the side length is 1 and ask how many of the diagonals they think they can work out the length of – and how they would work them out. Responses should include identifying right angled triangles and the use of Pythagoras' theorem.
- Give students a few minutes to discuss ideas in pairs and take feedback.
- Students then work in pairs to find the lengths of the diagonals.
- This is a good activity for students to engage with each other's mathematical thinking, demonstrating that many of the answers can be obtained in more than one way and that several expressions involving surds can be equivalent.

If necessary, at an appropriate point:

- Remind students that $(a+b)^2 = a^2 + b^2 + 2ab$ [PowerPoint slide 3]
- Ask how this might relate to $8 + 3 + 2\sqrt{8 \times 3}$
- Work through the simplification of $\sqrt{11 + 4\sqrt{6}} = \sqrt{8} + \sqrt{3}$ (outlined on the student help sheet). This will help students with the task. [PowerPoint slides 3 and 4]
- Give students the 3 questions to tackle if practice is needed

Hints:
Some diagonals are easier to work out than others, and some are a pre-requisite for others. Encourage students to start with something simple and work from there.
The order of simplicity is:

- Diagonal 2
- Diagonal 1
- Diagonal 5
- Diagonal 4
- Diagonal 3

Numbered in order of length, as below in the worked example.

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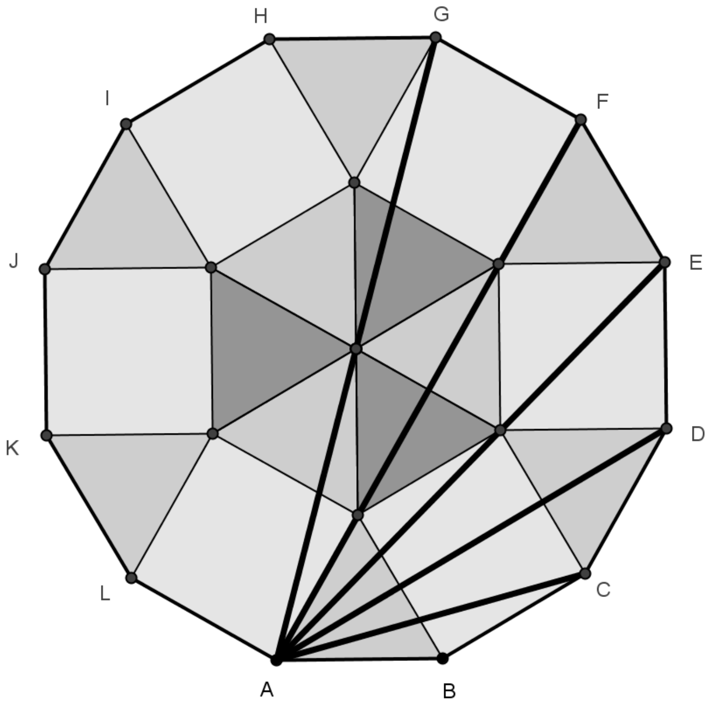
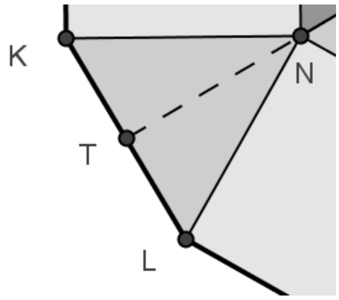
Worked example:
There are alternative ways to calculate many of these.
Key steps are shown rather than all stages of working.



AC: Diagonal 1
AD: Diagonal 2
AE: Diagonal 3
AF: Diagonal 4
AG: Diagonal 5

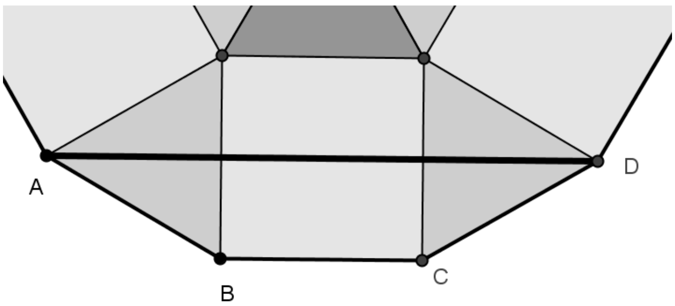


Firstly find the height of one of the equilateral triangles of side length 2.

Using Pythagoras' theorem:

$LN^2 = LT^2 + NT^2$
$NT = \sqrt{2^2 - 1^2}$
$\boldsymbol{NT = \sqrt{3}}$

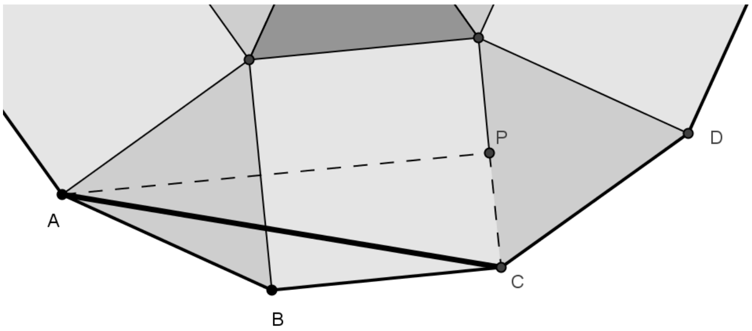
For Diagonal 2:



This diagonal is:

$\sqrt{3} + 2 + \sqrt{3} = \mathbf{2 + 2\sqrt{3}}$

For Diagonal 1:



$AP = 2 + \sqrt{3} \quad CP = 1$

Using Pythagoras' theorem:

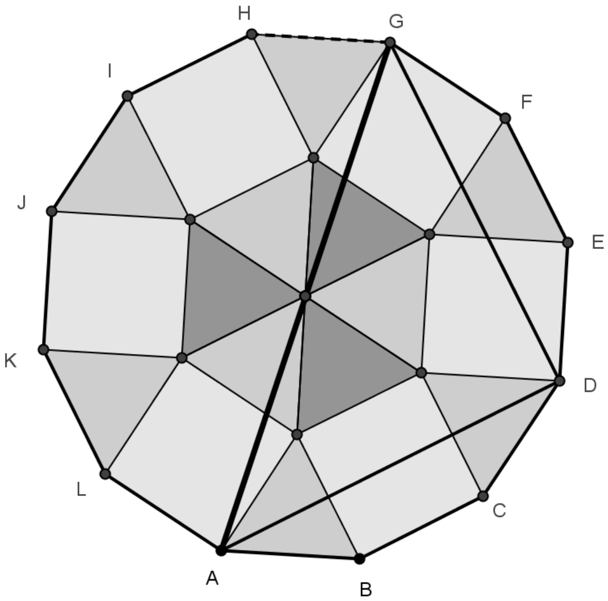
$AC^2 = AP^2 + CP^2$

$AC = \sqrt{(2 + \sqrt{3})^2 + 1^2}$

$AC = \sqrt{8 + 4\sqrt{3}}$

$\boldsymbol{AC = \sqrt{6} + \sqrt{2}}$

For Diagonal 5:



AG passes through the centre of the dodecagon, meaning that it is equivalent in length to 2 of diagonal AC, since each half of AG passes through exactly the same arrangement of a square and an equilateral triangle that AC does.

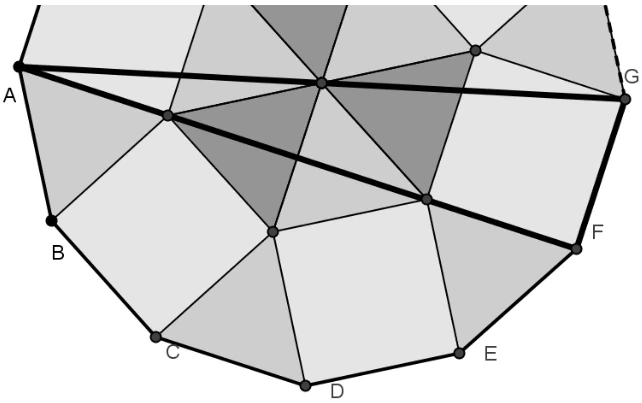
Therefore:

$\boldsymbol{AG = 2(\sqrt{6} + \sqrt{2})}$

Alternatively ADG is an isosceles right-angled triangle so AG can be calculated using Pythagoras' theorem.

This particular expression is a good one to use to demonstrate that there are several alternatives and can lead to discussions about what is meant by a 'simpler' expression.

For Diagonal 4:



AF is the height of two equilateral triangles plus the sides of two equilateral triangles.

$AF = 2 + \sqrt{3} + \sqrt{3} + 2$

$\boldsymbol{AF = 4 + 2\sqrt{3}}$

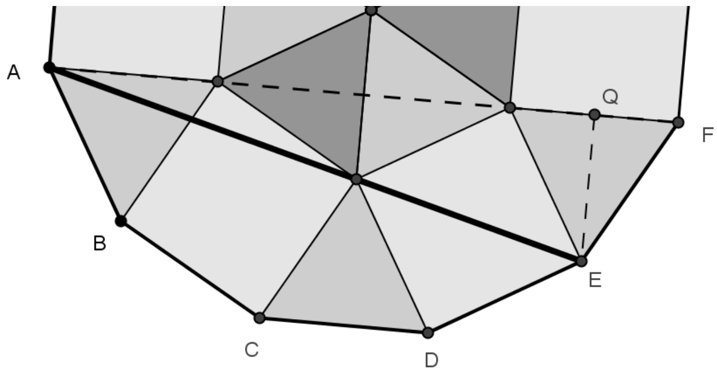
Alternatively, AFG is a right angle; AG is Diagonal 5.
Using Pythagoras' theorem:
$AG^2 = AF^2 + FG^2$

For Diagonal 3:

AF is Diagonal 4; AQ is therefore (the length of diagonal 4) – 1.

EQ is the perpendicular height of the equilateral triangle.

Using Pythagoras' theorem:



$AE^2 = AQ^2 + EQ^2$

$AE = \sqrt{(3 + 2\sqrt{3})^2 + (\sqrt{3})^2}$

$AE = \sqrt{24 + 12\sqrt{3}}$

$AE = \sqrt{18 + 6 + 2\sqrt{6 \times 18}}$

$AE = \sqrt{18} + \sqrt{6}$

$\boldsymbol{AE = 3\sqrt{2} + \sqrt{6}}$

An alternative for this calculation is to see AE as a combination of Diagonal 1 plus the diagonal of a square. i.e $AE = \sqrt{6} + \sqrt{2} + 2\sqrt{2}$